Use Pascal's Triangle to expand and simplify $(2n^4 - r^3)^5$. **SCORE: _____ / 5 PTS**

$$(2n^4)^5 + 5(2n^4)^4(-r^3) + 10(2n^4)^3(-r^3)^2 + 10(2n^4)^2(-r^3)^3 + 5(2n^4)(-r^3)^4 + (-r^3)^5$$

$$= \underbrace{32n^{20}}_{\frac{1}{2}} \underbrace{-80n^{16}r^3}_{1} \underbrace{+80n^{12}r^6}_{1} \underbrace{-40n^8r^9}_{1} \underbrace{+10n^4r^{12}}_{1} \underbrace{-r^{15}}_{\frac{1}{2}}$$

Using the formulae for the sums of the powers of integers in your textbook, find the sum $\sum_{n=1}^{310}(2n^2 - 5n + 3)$. **SCORE: _____ / 4 PTS**

Your final answer may **NOT** use … **NOR** $\sum$. It may use $+, -, \times, \div$. (It does **NOT** need to be simplified into a single number.)

$$2\sum_{n=1}^{310} n^2 - 5\sum_{n=1}^{310} n + \sum_{n=1}^{310} 3$$

$$= \underbrace{2 \cdot \frac{310(311)(621)}{6}}_{2} - \underbrace{5 \cdot \frac{310(311)}{2}}_{1\frac{1}{2}} + \underbrace{310 \cdot 3}_{\frac{1}{2}}$$

FJ started a 31 day treatment program which involved daily injections of a medication. The first day's injection was 18 mg, and each subsequent day's injection was 6% less than the previous day's injection. Find the total amount of the injections. **SCORE: _____ / 5 PTS**

Your final answer may **NOT** use ... **NOR** $\sum$. It may use $+, -, \times, \div$ and powers. (It does **NOT** need to be simplified into a single number.)

a_n = AMOUNT OF INJECTION ON n^{TH} DAY

$a_1 = 18$

$a_2 = 18(1-0.06) = 18(0.94)$

$a_3 = 18(0.94)^2$

$\vdots$

$a_n = 18(0.94)^{n-1}$

$$\text{TOTAL} = \frac{\overbrace{18}^{1}\overbrace{(1-0.94^{31})}^{2\frac{1}{2}}}{\underbrace{1-0.94}_{1\frac{1}{2}}}$$

Find the rational number representation of the repeating decimal $0.2\overline{63}$ using the method discussed in lecture. **SCORE: _____ / 6 PTS**
NOTE: Only the 63 is repeated.

$$\underline{0.2 + 0.063 + 0.00063 + 0.0000063 + \ldots} \quad 1$$

$$= \frac{2}{10} + \frac{0.063}{1 - \frac{1}{100}}$$

$$= \frac{2}{10} + \boxed{\frac{\frac{63}{1000}}{\frac{99}{100}}} \quad 2\tfrac{1}{2}$$

$$= \frac{2}{10} + \frac{63}{1000} \cdot \frac{100}{99}$$

$$= \boxed{\frac{2}{10}} + \boxed{\frac{7}{110}} \quad 1 \quad \tfrac{1}{2}$$

$$= \boxed{\frac{29}{110}} \quad 1$$

Use mathematical induction to prove that is 3 a factor of $n^3 + 6n^2 + 5n$ for all positive integers n. **SCORE: _____ / 10 PTS**

BASIS STEP: 3 IS A FACTOR OF $1^3 + 6(1)^2 + 5(1) = 12$ ①

MUST HAVE WORD "INTEGER"

INDUCTIVE STEP: ① ASSUME 3 IS A FACTOR OF $k^3 + 6k^2 + 5k$
FOR SOME PARTICULAR BUT ARBITRARY INTEGER ① $k \geq 1$ ①

[PROVE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 5(k+1)$]

$(k+1)^3 + 6(k+1)^2 + 5(k+1)$ ①

$$
\begin{aligned}
&= k^3 + 3k^2 + 3k + 1 \\
&\quad + 6k^2 + 12k + 6 \\
&\quad + 5k + 5 \\
&= (k^3 + 6k^2 + 5k) + (3k^2 + 15k + 12) \\
&= (k^3 + 6k^2 + 5k) + 3(k^2 + 5k + 4) \quad ②
\end{aligned}
$$

SINCE 3 IS A FACTOR OF BOTH $(k^3 + 6k^2 + 5k)$ AND $3(k^2 + 5k + 4)$ ①

THEREFORE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 5(k+1)$ ①

SO, 3 IS A FACTOR OF $n^3 + 6n^2 + 5n$ $\frac{1}{2}$
FOR ALL INTEGERS $n \geq 1$ $\frac{1}{2}$